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The Babylonian Right Triangle and Its Meanings by Pythagoras, Plato, and Proclus

The right triangle—or right-angled triangle—was an object of geometrical interest in cuneiform mathematical texts from southern Mesopotamia as early as the Old Babylonian Period (*c.* 20th to 16th centuries BCE). Admittedly, for practical calculations in geometrical drawings and cuneiform coefficient lists, the triangle's dimensions were sometimes rounded off in inexact ways. As scholars have long recognized, however, the Babylonians already knew of the so-called 'Pythagorean Rule'—as evident, for example, in stereotypical math problems involving a pole on horizontal ground leaning diagonally against a vertical wall. Famous cuneiform tablets such as 'YBC 7289' and especially 'Plimpton 322' relied with great precision on sets of 'Pythagorean triples'—a group of three numbers representing the width (w), the length (l), and the hypotenuse or diagonal (d) of a given right triangle.

Today, the Pythagorean Rule is popularly expressed using the formula $w^2 + l^2 = d^2$, which involves the square (^{2}) values of each side of a right triangle. This represented, even in antiquity, one of the best-known versions of the Rule—whose proof is strikingly demonstrated in Euclid's *Elements* (1.47), where the triangle's sides are treated not merely as numerical values raised to their second power (^{2}), but as borders of the areas of actual *square shapes*—but whose invention was ascribed to Pythagoras by later authors like Plutarch (*Non posse suaviter vivi* 11; *Quaestiones Convivales* 8.2.4), Diogenes Laertius (8.12), and Athenaeus (10.13). Importantly, Proclus' commentary on Euclid's Proposition mentions two additional and alternative methods for deriving Pythagorean triples: The first is assigned also to Pythagoras and begins with an odd number that is posited as the triangle's width, whereas a second procedure attributed to Plato is similar to the first but starts off with an even number.

I suggest how these alternative methods—in ways more direct than Euclid's Proposition—may be anchored in models and imagery original to Babylonian mathematics. By paying close attention to the particular forms of mathematical language and their conceptual models, I explore how the procedures credited to Pythagoras and Plato may have built upon older foundations, but reframed the problem in interesting new ways.

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Hero of Alexandria's *Metrica* offers a varied apparatus of problem-solving techniques for measuring the sizes of geometrical figures. The techniques and proofs in the work blend attributes of Greek geometry with elements of a tradition of practical mathematics that owes much to Mesopotamian and Egyptian mathematical traditions. For example, Hero combines, often in the same passage, the distinctive linguistic forms of Greek geometry (such as the use of third-person active imperatives and letter-labeled diagrams) with the very different features (like second-person active imperatives and numerical examples) that tend to keep studies of "practical mathematics" starkly separated from investigations of Greek geometry.

Rather than discarding the "practical" elements of the *Metrica* or viewing them as separate from the geometrical problems, Cuomo suggests a comparison with the linguistic model of "code-switching," where bilingual speakers shift from one language to another depending on the situation.[1] So rather than two mathematical traditions whose participants are largely siloed from one another, the *Metrica* might model a "bilingual" mathematical tradition that hybridizes the practical and geometrical traditions.

Cuomo's emphasis on "code-switching" is indeed a very productive lens through which to view the *Metrica* (and indeed other works, both by Hero and others). Still, many questions remain to be explored. How does the text actually serve to facilitate that work? What is Hero's purpose in including these features in his text? And what might all this tell us about "anchoring innovation" in Greek mathematics – in particular, is it always even possible to identify which "code" is the "anchor" and which is the "new"? Hero's approach offers the possibility of seeing "anchoring innovation" as a Janus-faced bootstrapping operation, where practical mathematics and geometry both serve in turn to anchor one another in weaving the fabric of an innovative hybrid mathematics.

In this paper, I sketch a two-pronged anchor in Hero's *Metrica* and a few related texts – let us label its prongs "cultural" and "cognitive" anchoring. "Cultural" anchoring practices link Hero's hybrid *Metrica* both to the Greek geometrical tradition and to an arithmetical tradition adapting Egyptian and perhaps Babylonian practices into a Greek (or Greco-Roman) context. These practices motivate and legitimate the knowledge developed in Hero's works back to cultural touchstones like Plato and Aristotle's musings on the justice effected by the geometrical mean, or the legendary antiquity of Egyptian traditions of land measurement. "Cognitive" anchoring, in turn, draws conceptual connections to help the reader build up comprehension of the text's content, rooting complex mathematical procedures in more elementary components (e.g. analogies between planar and solid figures) and cementing those connections through signposting markers in the text. The distinctive linguistic markers of the text strengthen these structural features for both types of anchor; they mark and reinforce generic conventions while simultaneously serving as what Clark calls "linguistic scaffolds."[2] I draw on work from cognitive science and mathematics pedagogy to explore how these "cognitive" and "cultural" anchors interact to produce texts that combine traditions productively to yield genuinely new ways of approaching mathematical problems.

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[1] Serafina Cuomo, "Mathematical Traditions in Ancient Greece and Rome," *HAU: Journal of Ethnographic Theory* 9, no. 1 (March 1, 2019): 80, https://doi.org/10.1086/703797.

[2] Andy Clark, *Supersizing the Mind: Embodiment, Action, and Cognitive Extension* (Oxford; New York: Oxford University Press, 2008), 44.

Teun Tieleman (Utrecht): ‘Galen’s Use of Hippocrates as an Anchor for Medical Innovation.’

In Geoffrey Lloyd’s apt words, Galen’s Hippocratism is so familiar to us that it is easy to take it too much for granted. Galen’s choice of Hippocrates as the authority of the past whom he admires and says to follow actually raises a few pressing questions, just as Galen’s interpretation of Hippocrates. Building on the relevant studies by Lloyd, Wesley D. Smith and others I will examine Galen’s use of Hippocrates for bestowing legitimacy upon his medical doctrines, methods and practices, especially when these were (and could be rejected as) innovative. Can we understand this use in terms of the notion of anchoring what is new in what is old and familiar? A striking example is provided by Galen’s attempt to (re)introduce anatomical research as an integral part of medicine, a controversial issue among doctors since Hellenistic times. I will focus on Galen’s from a modern historiographical perspective highly disputable attribution of anatomical knowledge and the practice of dissection to Hippocrates and his circle in the preface to the second book of his comprehensive handbook *Anatomical Procedures* (II 280-282 K.). Does this passage reflect the need for Galen to anchor novel and controversial proposals? And does the picture emerging from it fit with how we find him using Hippocrates in other contexts? What does his use of Hippocrates tell us about his idea of progress in medicine and science in general?

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Naturalizing medical prognosis in the *Prometheus Bound*

On a spring day in the fifth century BCE, Athenian spectators gathered in the theater of Dionysos to experience three connected tragedies dramatizing the relation between the Titan Prometheus, fire-giver and inventor of all human technologies, and the supreme god, Zeus. While our single extant play, *Prometheus Bound*, focuses on the intense conflict between the antagonists, fragments from the *Prometheus Unbound* suggest that the trilogy ended in a reconciliation, with Prometheus's bondage being both compensated and remembered for through the dedication of torch-races and his wearing of a crown.

I propose to read *Prometheus Bound* as a dramatization, through the medium of Athenian tragedy, of the dialectics of "anchoring innovation" and the challenge of articulating what is perceived as new and potentially disruptive into the pre-existing order. Drawing on critical concepts developed by Marxist critics Antonio Gramsci, Fredric Jameson, and Raymond Williams, I read *Prometheus Bound* as a symbolic resolution of contradictions among fifth-century responses to technological innovation, and I show how the culturally dominant form of Athenian tragedy was alert and responsive to alternatives and oppositions that questioned or threatened hegemony.

Although the extant fragments from the *Prometheus Unbound* offer only tantalizing glimpses into the process of Prometheus' integration within the rule of Zeus, I argue that the extant play exemplifies the process of conceptual anchoring of a specific technique, medical prognosis, through a narrative of disease and cure. Shortly after Prometheus' triumphant catalogue of his technological gifts, culminating on drugs for *all* diseases, the mad Io, part cow, part maiden, erupts on stage and asks him for a cure. Rather than healing her himself, however, Prometheus delivers a long prophecy culminating with her cure in Egypt through the magical touch of Zeus. Io's story interacts with Prometheus' medical art in complex ways. While her departure in a state of frenzy undermines his previous claim about the infallibility of pharmacological medicine, his ability to foresee the time, location, and means of her cure parallels a rising development in Hippocratic medicine—prognosis, or the ability to providing patients with a narrative of their past, present, and future.

Through a contextual analysis of the three forms of healing that come into play in Io's story—pharmacology, divine touch, and prognosis—, I show how the narrative anchors the newer and potentially threatening art of prognosis into the dominant rule of Zeus. The paper falls into three parts. The first part sets up the ideological background against which *Prometheus Bound* unfolds by analyzing the role of prognosis in the self-fashioning of Hippocratic medicine as a reliable *technê*. The second section shows how Prometheus' prophecies may be read as a form of prognosis, and how they rely on beliefs about the regularity of natural laws that implicitly challenge the hegemonic notion of an omnipotent Zeus. The third part analyzes rhetorical

means that integrate and naturalize Promethean prognosis into Io's story, including the deployment of pharmacological medicine as its foil, and its subordination to the telos of Zeus' touch. Taken as a whole, the paper provides a case study for the anchoring of a distinctive feature of Hippocratic medicine through dramatic performance.

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ABSTRACT

Risky Business: Anchoring Blown Glass and *Terra Sigillata* Production in the Face of Risk

Innovation is a risky business; in craft production especially, innovation requires investment in new technologies without the certainty of recovering that investment. This reality, crucial for understanding the lived experience of the innovative process, has, however, not been consistently recognized by scholars of ancient technology, who have had to focus their efforts on refuting the idea that technology was stagnant in the Classical world (Finley 1965). Research has now succeeded in establishing the pervasive nature of innovation, especially in the Roman world (e.g. Greene 2000; numerous contributions in Oleson, ed. 2008), and has laid the groundwork for studies of technology in its social context, which to-date have primarily focused on the impact of innovations in the economy (e.g. Wilson 2002) and daily life (e.g. Flohr 2016). We are therefore now well-placed to move the debate forward by examining *how* innovation occurred, a need that becomes apparent when the risk inherent in technological innovation is recognized. How were inventions were taken up and spread? What factors facilitated the anchoring of new technologies within the context of established production techniques?

A combination of ethnographic and theoretical work suggests that perception of risk (i.e. as good or bad) is a key variable in making the decision to innovate (van der Leeuw 1989; Papousek 1989). Thus, to answer the 'how' of innovation, it becomes necessary to determine what factors might have ameliorated the perceived risk, for it is in those that we will find our anchors.

Taking its cue from these considerations, this paper explores blown glass production and the production of *terra sigillata* and African Red Slip in an attempt to shed light on the lived processes of innovation in the Roman world and the anchoring factors that facilitated technological innovation. Examination of the relevant productive processes reveals that the risks associated with innovation lay primarily with the investment in new, unproven techniques (e.g. indirect firing of *terra sigillata*) and the associated costs in time and materials (Cuomo di Caprio 2007; Larson 2019). However, analysis of archaeological materials associated with production, specifically cullet deposits and kiln furniture, along with the more general processes of innovation, reveals that both material and social factors played decisive roles in ameliorating the perception of risk, anchoring blown glass and *terra sigillata* in established technological practices and cultural habits, and overall facilitating their spread and adoption. The avenues of investigation opened by the paper's findings – into areas as diverse as the daily lives and decision-making process of craftspeople, the integration of the Mediterranean, and the formation of a Roman 'consumer culture' – indicate just how imperative it is to consider innovation as a process and to study it within its social context.

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From Hand-Bows to Artillery

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The paper will discuss the transformation of hand-bows into artillery through two parameters: 1) how these inventions were spread and promoted by being anchored to needs of practical nature; and, at the same time, 2) how they were rejected since they redefined social values, specifically *andreia*, and thus, threatened to substitute for humanity. The paper will first discuss the case of the hand-bow as discussed mainly in Euripides' *HF*. The bow in this play is held responsible for helping Heracles to achieve the cultural objective of *andreia*. However, at the same time, the bow, along with its technical properties,[1] enflames a discussion on cowardice (157-203). Specifically, according to Lycus, if Heracles is ever deprived of the bow, he will not only lose the title of the bravest man, but, in addition, he will prove to be merely a coward; thus, Heracles, the archer, without his bow and arrows, could not be victorious in the battle as he has never learned how to fight with the spear and shield, that is, he never experienced a face-to-face encounter with the enemy. The paper will then explore the case of *gastrophetes* or "belly bows" as attested in Biton's *Construction of War Machines and Artillery* 65-7 and in Heron's *Belopoeica* 75-81 (ed. Mardsen), where instead of using the hand, the operator of this device could lean this on his belly; this allows him to increase the pulling power and thus to have a better result. Finally, the paper will examine artillery machines (mainly in Hero's and Philon's *Belopoeica*) which by utilizing torsion (instead of tension) increase even more their power and range and thus, promote their quicker spread and adoption;[2] again, however, these devices are being criticised since they cancel any relationship between the device and the physical hand/body of the warrior, and thus, threaten to turn the latter into a simple operator (see Plut. *Mor*. 191e).

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[1] According to Amphitryon, the bow offers abundant solutions, for instance, if an arrow is lost, it can rapidly be replaced by countless other arrows and it protects the safety of the warrior etc.
[2] See Cuomo 2002, 169, "an increasing number of artificial elements are interposed between the hand and the projectile itself".